

Name key

3. A hypothetical quantum-mechanical system has the energy levels  $\epsilon = ak(k+2)$  where  $k$  is any positive integer and  $a$  is positive. If the selection rule for absorption is  $\Delta k = +2$

(a) What is the expression for the possible energies at which light can be absorbed?

(b) What is the energy spacing [i.e.,  $\Delta(\Delta\epsilon)$ ] between two adjacent lines? (Consider  $k_{ini}$  for the second line to be  $1+k_{ini}$  for the first line).

$$\begin{aligned} a/ \quad \Delta\epsilon &= a \left[ k_{fin} (k_{fin} + 2) - k_{ini} (k_{ini} + 2) \right] \\ &= a \left[ (k_{ini} + 2) (k_{ini} + 2 + 2) - k_{ini} (k_{ini} + 2) \right] \\ &= a \left[ k_{ini}^2 + 6k_{ini} + 8 - k_{ini}^2 - 2k_{ini} \right] \\ &= a \left[ 4k_{ini} + 8 \right] = 4a \left[ k_{ini} + 2 \right] \end{aligned}$$

$$\begin{aligned} b/ \quad \Delta(\Delta\epsilon) &= 4a \left[ (k_{ini_2} + 2) - (k_{ini_1} + 2) \right] \\ &= 4a \left[ k_{ini_2} + 1 - k_{ini_1} \right] \\ &= 4a \end{aligned}$$